$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}; g^{\mu\nu} \longrightarrow \tilde{g}^{\mu\nu} = \Omega^{-2} g^{\mu\nu}; \phi \longrightarrow \tilde{\phi} = \Omega^{-\frac{1}{3}}\phi$$

$$\tilde{S}_{\mu\nu}{}^{\lambda\delta} = \underbrace{\tilde{\phi}^2 \tilde{R}_{\mu\nu}{}^{\lambda\delta}}_{\boxed{0}} \underbrace{+k_1 \delta^{[\lambda}_{[\mu}\delta^{\delta]}_{\nu]} \tilde{\nabla}_\rho \tilde{\phi} \tilde{\nabla}^\rho \tilde{\phi}}_{\boxed{1}} \underbrace{+k_2 \tilde{\phi} \delta^{[\lambda}_{[\mu} \tilde{\nabla}_{\nu]} \tilde{\nabla}^{\delta]} \tilde{\phi}}_{\boxed{2}} \underbrace{+k_3 \delta^{[\lambda}_{[\mu} \tilde{\nabla}_{\nu]} \tilde{\phi} \tilde{\nabla}^{\delta]} \tilde{\phi}}_{\boxed{3}}$$

$\boxed{0}$

$$\tilde{R}_{ab}{}^{cd} = \Omega^{-2}\left\{R_{ab}{}^{cd} - 4\delta^{[c}_{[a}\nabla_{b]}\nabla^{d]} ln\Omega + 4\delta^{[c}_{[a}\nabla_{b]} ln\Omega \nabla^{d]} ln\Omega - 2\delta^{c}_{[a}\delta^{d}_{b]}\nabla_e ln\Omega \nabla^e ln\Omega\right\}$$

$$\tilde{\phi}^2 \tilde{R}_{ab}{}^{cd} = \Omega^{-\frac{8}{3}}\phi^2 \Big(R_{ab}{}^{cd} - 4\delta^{[c}_{[a}\nabla_{b]}\nabla^{d]} ln\Omega + 4\delta^{[c}_{[a}\nabla_{b]} ln\Omega \nabla^{d]} ln\Omega$$
$$- 2\delta^{c}_{[a}\delta^{d}_{b]}\nabla_e ln\Omega \nabla^e ln\Omega\Big)$$

■

$\boxed{1}$

$$\tilde{\nabla}_\rho \tilde{\phi} = \nabla_\rho \left(\Omega^{-\frac{1}{3}}\phi\right) = \Omega^{-\frac{1}{3}}\nabla_\rho \phi + \phi \nabla_\rho \Omega^{-\frac{1}{3}} = \Omega^{-\frac{1}{3}}\left(\nabla_\rho \phi - \frac{1}{3}\phi \nabla_\rho ln\Omega\right)$$

$$\tilde{\nabla}^\rho \tilde{\phi} = \tilde{g}^{\rho\sigma}\tilde{\nabla}_\sigma \tilde{\phi} = \Omega^{-\frac{1}{3}}\tilde{g}^{\rho\sigma}\left(\nabla_\sigma \phi - \frac{1}{3}\phi \nabla_\sigma ln\Omega\right) = \Omega^{-\frac{7}{3}}\left(\nabla^\rho \phi - \frac{1}{3}\phi \nabla^\rho ln\Omega\right)$$

$$\tilde{\nabla}_\rho \tilde{\phi} \tilde{\nabla}^\rho \tilde{\phi} = \Omega^{-\frac{8}{3}}\left(\nabla_\rho \phi - \frac{1}{3}\phi \nabla_\rho ln\Omega\right)\left(\nabla^\rho \phi - \frac{1}{3}\phi \nabla^\rho ln\Omega\right)$$

$$= \Omega^{-\frac{8}{3}}\left(\nabla_\rho \phi \nabla^\rho \phi - \frac{2}{3}\phi \nabla_\rho ln\Omega \nabla^\rho \phi + \frac{1}{9}\phi^2 \nabla_\rho ln\Omega \nabla^\rho ln\Omega\right)$$

$$k_1 \delta^{[c}_{[a}\delta^{d]}_{b]} \tilde{\nabla}_e \tilde{\phi} \tilde{\nabla}^e \tilde{\phi} = \Omega^{-\frac{8}{3}} k_1 \delta^{[c}_{[a}\delta^{d]}_{b]} \left(\nabla_e \phi \nabla^e \phi - \frac{2}{3}\phi \nabla_e ln\Omega \nabla^e \phi + \frac{1}{9}\phi^2 \nabla_e ln\Omega \nabla^e ln\Omega\right)$$

■

$\boxed{2}$

$$C^d{}_{be} = \delta^d_b \nabla_e ln\Omega + \delta^d_e \nabla_b ln\Omega - g_{be}\nabla^d ln\Omega$$

$$\delta^c_a \tilde{\nabla}_b \tilde{\nabla}^d \tilde{\phi} = \delta^c_a \nabla_b (\tilde{\nabla}^d \tilde{\phi}) + \delta^c_a C^d{}_{be} \tilde{\nabla}^e \tilde{\phi}$$

$$= \delta^c_a \nabla_b \left\{\Omega^{-\frac{7}{3}}\left(\nabla^d \phi - \frac{1}{3}\phi \nabla^d ln\Omega\right)\right\} + \delta^c_a C^d{}_{be} \Omega^{-\frac{7}{3}}\left(\nabla^e \phi - \frac{1}{3}\phi \nabla^e ln\Omega\right)$$

$$= \delta^c_a \nabla_b \left(\Omega^{-\frac{7}{3}}\right)\left(\nabla^d \phi - \frac{1}{3}\phi \nabla^d ln\Omega\right) + \Omega^{-\frac{7}{3}}\delta^c_a \nabla_b \left(\nabla^d \phi - \frac{1}{3}\phi \nabla^d ln\Omega\right)$$
$$+ \delta^c_a C^d{}_{be} \Omega^{-\frac{7}{3}}\left(\nabla^e \phi - \frac{1}{3}\phi \nabla^e ln\Omega\right)$$

$$= -\frac{7}{3}\Omega^{-\frac{7}{3}}\delta^c_a \nabla_b ln\Omega \left(\nabla^d \phi - \frac{1}{3}\phi \nabla^d ln\Omega\right)$$
$$+ \Omega^{-\frac{7}{3}}\delta^c_a \left(\nabla_b \nabla^d \phi - \frac{1}{3}\nabla_b \phi \nabla^d ln\Omega - \frac{1}{3}\phi \nabla_b \nabla^d ln\Omega\right)$$
$$+ \delta^c_a C^d{}_{be} \Omega^{-\frac{7}{3}}\left(\nabla^e \phi - \frac{1}{3}\phi \nabla^e ln\Omega\right)$$

$$= \Omega^{-\frac{7}{3}}\Big(\delta_a^c\nabla_b\nabla^d\phi - \frac{4}{3}\delta_a^c\nabla_b ln\Omega\nabla^d\phi - \frac{4}{3}\delta_a^c\nabla^d ln\Omega\nabla_b\phi + \frac{7}{9}\phi\delta_a^c\nabla_b ln\Omega\nabla^d ln\Omega$$

$$- \frac{1}{3}\phi\delta_a^c\nabla_b\nabla^d ln\Omega + \delta_a^c\delta_b^d\nabla_e ln\Omega\nabla^e\phi - \frac{1}{3}\phi\delta_a^c\delta_b^d\nabla_e ln\Omega\nabla^e ln\Omega\Big)$$

$k_2\tilde{\phi}\delta_{[a}^{[c}\tilde{\nabla}_{b]}\tilde{\nabla}^{d]}\tilde{\phi}$

$$= \Omega^{-\frac{8}{3}}k_2\phi\left(\delta_{[a}^{[c}\nabla_{b]}\nabla^{d]}\phi - \frac{4}{3}\delta_{[a}^{[c}\nabla_{b]}ln\Omega\nabla^{d]}\phi - \frac{4}{3}\delta_{[a}^{[c}\nabla^{d]}ln\Omega\nabla_{b]}\phi\right.$$

$$+ \frac{7}{9}\phi\delta_{[a}^{[c}\nabla_{b]}ln\Omega\nabla^{d]}ln\Omega - \frac{1}{3}\phi\delta_{[a}^{[c}\nabla_{b]}\nabla^{d]}ln\Omega + \delta_{[a}^{[c}\delta_{b]}^{d]}\nabla_e ln\Omega\nabla^e\phi$$

$$\left.- \frac{1}{3}\phi\delta_{[a}^{[c}\delta_{b]}^{d]}\nabla_e ln\Omega\nabla^e ln\Omega\right)$$

■

$\boxed{3}$

$$\tilde{\nabla}_b\tilde{\phi}\tilde{\nabla}^d\tilde{\phi} = \Omega^{-\frac{8}{3}}\left(\nabla_b\phi - \frac{1}{3}\phi\nabla_b ln\Omega\right)\left(\nabla^d\phi - \frac{1}{3}\phi\nabla^d ln\Omega\right)$$

$$= \Omega^{-\frac{8}{3}}\Big(\nabla_b\phi\nabla^d\phi - \frac{1}{3}\phi\nabla^d ln\Omega\nabla_b\phi - \frac{1}{3}\phi\nabla_b ln\Omega\nabla^d\phi$$

$$+ \frac{1}{9}\phi^2\nabla_b ln\Omega\nabla^d ln\Omega\Big)$$

$k_3\delta_{[a}^{[c}\tilde{\nabla}_{b]}\tilde{\phi}\tilde{\nabla}^{d]}\tilde{\phi}$

$$= \Omega^{-\frac{8}{3}}k_3\left(\delta_{[a}^{[c}\nabla_{b]}\phi\nabla^{d]}\phi - \frac{1}{3}\phi\delta_{[a}^{[c}\nabla^{d]}ln\Omega\nabla_{b]}\phi - \frac{1}{3}\phi\delta_{[a}^{[c}\nabla_{b]}ln\Omega\nabla^{d]}\phi\right.$$

$$\left.+ \frac{1}{9}\phi^2\delta_{[a}^{[c}\nabla_{b]}ln\Omega\nabla^{d]}ln\Omega\right)$$

■

$\boxed{0} + \boxed{1} + \boxed{2} + \boxed{3}$

$$\tilde{\phi}^2 \tilde{R}_{ab}{}^{cd} + k_1 \delta^{[c}_{[a} \delta^{d]}_{b]} \tilde{\nabla}_e \tilde{\phi} \tilde{\nabla}^e \tilde{\phi} + k_2 \tilde{\phi} \delta^{[c}_{[a} \tilde{\nabla}_{b]} \tilde{\nabla}^{d]} \tilde{\phi} + k_3 \delta^{[c}_{[a} \tilde{\nabla}_{b]} \tilde{\phi} \tilde{\nabla}^{d]} \tilde{\phi}$$

$$\begin{aligned}
= \Omega^{-\frac{8}{3}} \Big(& \phi^2 R_{ab}{}^{cd} - 4\phi^2 \delta^{[c}_{[a} \nabla_{b]} \nabla^{d]} ln\Omega + 4\phi^2 \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} ln\Omega \\
& - 2\phi^2 \delta^{c}_{[a} \delta^{d}_{b]} \nabla_e ln\Omega \nabla^e ln\Omega + k_1 \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e \phi \nabla^e \phi \\
& - \frac{2k_1}{3} \phi \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e \phi + \frac{k_1}{9} \phi^2 \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e ln\Omega \\
& + k_2 \phi \delta^{[c}_{[a} \nabla_{b]} \nabla^{d]} \phi - \frac{4k_2}{3} \phi \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} \phi - \frac{4k_2}{3} \phi \delta^{[c}_{[a} \nabla^{d]} ln\Omega \nabla_{b]} \phi \\
& + \frac{7k_2}{9} \phi^2 \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} ln\Omega - \frac{k_2}{3} \phi^2 \delta^{[c}_{[a} \nabla_{b]} \nabla^{d]} ln\Omega \\
& + k_2 \phi \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e \phi - \frac{k_2}{3} \phi^2 \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e ln\Omega \\
& + k_3 \delta^{[c}_{[a} \nabla_{b]} \phi \nabla^{d]} \phi - \frac{k_3}{3} \phi \delta^{[c}_{[a} \nabla^{d]} ln\Omega \nabla_{b]} \phi - \frac{k_3}{3} \phi \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} \phi \\
& + \frac{k_3}{9} \phi^2 \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} ln\Omega \Big)
\end{aligned}$$

$$\begin{aligned}
= \Omega^{-\frac{8}{3}} \Big(& \phi^2 R_{ab}{}^{cd} + k_1 \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e \phi \nabla^e \phi + k_2 \phi \delta^{[c}_{[a} \nabla_{b]} \nabla^{d]} \phi + k_3 \delta^{[c}_{[a} \nabla_{b]} \phi \nabla^{d]} \phi \\
& - 2\phi^2 \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e ln\Omega - \frac{12 + k_2}{3} \phi^2 \delta^{[c}_{[a} \nabla_{b]} \nabla^{d]} ln\Omega \\
& + \frac{k_1 - 3k_2}{9} \phi^2 \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e ln\Omega \\
& + \frac{36 + 7k_2 + k_3}{9} \phi^2 \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} ln\Omega - \frac{4k_2 + k_3}{3} \phi \delta^{[c}_{[a} \nabla_{b]} ln\Omega \nabla^{d]} \phi \\
& - \frac{4k_2 + k_3}{3} \phi \delta^{[c}_{[a} \nabla^{d]} ln\Omega \nabla_{b]} \phi + \frac{3k_2 - 2k_1}{3} \phi \delta^{[c}_{[a} \delta^{d]}_{b]} \nabla_e ln\Omega \nabla^e \phi \Big)
\end{aligned}$$ [1]

$$k_1 = -18; k_2 = -12; k_3 = 48$$

$$S_{\mu\nu}{}^{\lambda\delta} = \phi^2 R_{\mu\nu}{}^{\lambda\delta} - 18 \delta^{[\lambda}_{[\mu} \delta^{\delta]}_{\nu]} \nabla_\rho \phi \nabla^\rho \phi - 12 \phi \delta^{[\lambda}_{[\mu} \nabla_{\nu]} \nabla^{\delta]} \phi + 48 \delta^{[\lambda}_{[\mu} \nabla_{\nu]} \phi \nabla^{\delta]} \phi$$

1

$$\delta^{[c}_{[a} \delta^{d]}_{b]} = \frac{1}{2} \left(\delta^{c}_{[a} \delta^{d}_{b]} - \delta^{d}_{[a} \delta^{c}_{b]} \right) = \frac{1}{4} \left(\delta^c_a \delta^d_b - \delta^c_b \delta^d_a - \delta^d_a \delta^c_b + \delta^d_b \delta^c_a \right) = \frac{1}{2} \left(\delta^c_a \delta^d_b - \delta^c_b \delta^d_a \right) = \delta^{c}_{[a} \delta^{d}_{b]}$$